

MathNow

(Grades 1 – 3)

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Grades 1 – 8, 2005 (Revised)

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Trillium Listed

Estimation

by Ron Sauer

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1. Estimation

As noted in an earlier document, computational estimation and mental math are receiving added attention in mathematics curriculum guidelines and teacher resources. This is also true of estimation (sometimes called holistic or perceptual estimation). While computational estimation relies on the use of operational strategies (e.g., rounding, front-end), estimation relies on a set of different strategies and principles. Despite the differences, these skills and concepts are certainly connected through the ways in which they promote number sense.

Developing a growing comfort with numbers takes practice and exposure to all kinds of settings in which estimation is sufficient and desirable. As Barry Onslow has noted,

“Estimating develops a sense of number. The skill is developed through practice and reflection. The more we do it, and the more we think about why we do it, the better we become. To be successful children need many opportunities to practice estimation and to learn from previous experiences.”
(Onslow 2002)

1.1 Some Basic Principles of Estimation

- a) Estimation is often used in measurement activities, and in tasks that require a sense of quantity.
- b) Measurement activities for most grades include the use of standard and non-standard units
For example:
 - About how many skipping rope lengths is it from the side door to the drinking fountain?
 - About how many metres is it from our classroom to the resource centre?
 - About how many football fields placed end for end would match the height of the CN Tower?
- c) Students need to develop their own perceptual anchors (or comparison points/referents) for estimating. (A perceptual anchor is a quantity whose size is readily perceived in different settings. Thus a child can use these anchors to make comparative estimates of other quantities.) Carlow, 1986
For example:
 - If your finger is about 1 cm wide, about how many centimetres long is this book?
 - There are 10 stamps in the first row. About how many stamps (e.g., 16) are in the second row?
- d) While it is common to see estimation activities where students estimate, then check by measuring, this practice has received a mixed review in the literature. Carlow (1986) notes that that this idea of estimate then check quickly leads to the perception that estimating is not important.
There are times, however, when the process of estimate then measure is appropriate. This is particularly true when the focus of the process is to help students validate and refine their estimation skills. That said, students still need lots of opportunities to work in settings where a simple estimate is sufficient.
- e) In every day life it is often estimation rather than computation that gives us answers to questions such as:
 - Do I have enough cash to buy both bread and milk?
 - Is there enough gas in the car to get to Sudbury?
 - How many fans were in the Sky Dome last night?

1.2 Some General Samples of Estimation in Measurement

a) Linear

- About how many loonies would it take to stretch across one side of your desk to the other?
- About how many toothpicks wide is your desk?
- Find something in the classroom that is about 1 m long.
- Draw a straight line that is about 15 cm long.
- How many strides long is the distance from the bookcase to the door?
- What places are about 1 km from your home?

b) Perimeter

- About how many centimetres (toothpicks, straws, playing cards, erasers) are needed to measure the perimeter of the doormat?
- About how many metres (skipping ropes, body lengths, etc.) is the perimeter of the gym floor?

c) Area

- About how many centimetre cubes would it take to cover this piece of paper?
- About how many \$5 bills would it take to cover your desk?
- About how many of these square stickers would it take to cover the shoe box lid?
- Estimate the area of material needed to make a shirt.

d) Capacity

- About how many baby food jars would it take to fill this container?
- What containers in this room would hold about one litre?
- About how many of these little bottles (jars, glasses, cups) would you need to hold all of the orange juice in this jug?
- About how many litres of water would the wastepaper basket hold?

e) Volume

- About how many blocks would it take to fill this box?
- About how many soccer balls (dinosaurs, pop machines, dogs, whales) do think would fit in this room?
- About how many students could fit inside a model of a cubic metre?

f) Mass

- About how many wooden blocks would be needed to balance this scale with an apple?
- Hold a book from the library. Do you think it weighs more or less than 500 g?
- Which do you think is heavier: the pumpkin or the squash?
- Estimate the mass of each, then place the cucumber, the grapefruit, the zucchini, and the baking potato in order from the lightest to the heaviest.

g) Temperature (establishing referents or anchors)

- The water in this glass is freezing. It must be 0°C or less.
- This room seems comfortable so it must be about 20°C

h) Time

- About how many times will your heart beat in 1 hour?
- Estimate the total time given for commercials during one hour of TV.
- About how far could you walk in 15 minutes?
- About how long would it take you to write the numbers from 1 to 200?
- Close your eyes. Open them after you think 30 seconds has passed. How close was your estimate?
- How long will this birthday candle burn?

1.3 Estimation Questions that Require Investigation

Estimation questions of this type (sometimes referred to as Fermi problems) are often so intriguing that they lead to interesting investigations. Joan and Marc Ross have noted that they can be solved by a series of estimates. These tasks also promote number sense and problem solving.

For example:

What is your estimate of

- the number of ice cubes in an ice cube tray?
- the mass of a basketball?
- the number of parking meters in your city/town?
- the number of spokes in an average bicycle wheel?
- the number of people named Smith in your telephone book?
- the average life span of a housefly?
- the length of the longest street in your city/town?

One characteristic [of Fermi problems] is that most people can be led to a solution through a series of simple steps that use only common sense and numbers that are either generally known or amenable to estimation. (Ross, NCTM, p. 175)

1.4 Some Sampling Strategies

In the following examples, students look for a representative sample or anchor, then use it to estimate the total number of objects.

a) Stars

- A rectangle contains pictures of 50-60 randomly placed small stars. Ask “About how many stars are there in the rectangle?”
- Students might subdivide the rectangle into 4 quadrants, count or “guesstimate” the number in one quadrant, then multiply by four. Another strategy is to circle a cluster of stars, then, using this as an area anchor, determine about how many of these clusters would cover the rectangle.

b) Cookie Jar

- “Estimate how many Oreo cookies (candies, marbles) there are in this glass jar.”
- Students might estimate the number of items in one layer, then multiply by the number of layers.

c) Cat

- The outline of a cat is drawn on a piece of grid paper. Ask, “about how many square units is the area of the picture?”
- Students might mentally divide the area into rectangles, quickly count the average number of square units in each rectangle, then find the sum of these areas.

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2. Computational Estimation

As Robert Reys has noted, “estimation has seldom received serious attention in curriculum materials, despite its importance.” In recent years, however, there has been increasing interest in estimation. Curriculum developers, textbook authors, and classroom teachers have begun to acknowledge the importance of estimation, both in classroom settings and in the real world. A large portion of the computation done by adults in everyday life involves some form of mental math or mental computation. Adults use the context of a particular problem or situation to decide whether an exact answer is required, or if an estimate is sufficient. Recent classroom materials increasingly provide opportunities for students to develop the skills needed to become successful in using mental math strategies for estimation, or for producing exact answers.

2.1 Some Basic Principles of Computational Estimation

- a) Estimation is done quickly and mentally, and should produce reasonable answers.
- b) Estimation may be all that is required in a particular context.
- c) Estimations within a reasonable range that is often determined by the context, are considered acceptable.
- d) Estimates can be adjusted or refined more precisely to produce an answer closer to the exact answer.
- e) Estimation can be used to judge the reasonableness of an exact answer obtained with a calculator or with paper-and-pencil.

2.2 Some Basic Features of Computational Estimation

- a) Estimation requires that students choose from a set of words in order to describe, report, and communicate results (e.g., about, around, approximately, almost close to, closer, over, a little more (less) than, under, etc.).
- b) Estimation requires practice with some important related skills.

For example:

- i. Proficiency with basic facts
- ii. Mentally adding multiples of 10, 100, 1000
 - $50 + 30$, $200 + 500$, $2000 + 300 + 200$
- iii. Mentally multiplying powers and multiples of 10, 100, 1000
 - 7×2 , 7×20 , 7×200
 - 5×10 , 5×100 , 5×1000
 - 40×70 , 40×700 , 40×7000
- c) Estimation requires that students interpret the context of a problem (i.e., is an estimate sufficient?)
- d) Estimation requires that students have proficiency with place value concepts and skills, and use that knowledge to confidently decompose numbers (take them apart) and compose numbers (re-assemble them.)

2.3 Some General Strategies for Computational Estimation

2.3.1 Addition

Front-end Strategy:

Front-end estimation focuses on computing the sum of the lead digits. The estimate can be refined or adjusted by finding the sum of the next column. (The decision to use an adjustment is generally influenced by the context or situation.)

For example:

$$\begin{array}{r} 247 \\ 328 \\ + 130 \\ \hline \end{array}$$

- Add front end: $200 + 300 + 100 = 600$ Estimate: $600 +$

(NOTE: The “+” sign is used in some, but not all, math materials to denote that 600 is an underestimate, thus, further estimates must be adjusted upwards, or “something else has to be added to the estimate”. In other words, the actual answer is greater than the estimate. (Whether or not we use this symbol and its attendant meanings will depend on how much you think it will help or confuse kids.)

- Do you want it closer?

Adjust Up; Add next column: $40 + 20 + 30 = 90$, Closer Estimate: $690 +$

a) Rounding Strategy:

Rounding strategy focuses on the use of a set of specific rules to round numbers up or down to the nearest 10, 100, 1000, etc.

For example:

$$\begin{array}{r} 284 \\ 214 \\ + 430 \\ \hline \end{array}$$

- Round numbers up or down to nearest 10, 100, 1000, etc. (284 rounds up to 300; 214 rounds down to 200; and 430 rounds down to 400)
- Add the rounded numbers $300 + 200 + 400 = 900$

Estimate: 900

b) Grouping for “Nice Numbers” Strategy:

Grouping focuses on organizing numbers into groups of “nice numbers” that are easy to calculate mentally.

For example:

$$\begin{array}{r} 467 \\ 326 \\ 381 \\ + 217 \\ \hline \end{array}$$

- Add front end: $400 + 300 + 300 + 200 = 1200$ Estimate: $1200 +$
- Do you want it closer?

Group numbers: $67 + 26$ is about **100**; $81 + 17$ is about **100** Closer Estimate: 1400

c) Clustering Strategy:

Clustering can be used when a group of numbers centres (clusters) around an average number.

For example:

$$85 + 76 + 79 + 81 + 82 + 75 + 78$$

Clusters around 80, so $7 \times 80 = 560$ Estimate: 560

2.3.2 Subtraction

a) Front-end Strategy:

Front-end estimation focuses on computing the difference of the lead digits. The estimate can be refined or adjusted by finding the difference of the next column.

For example:

$$\begin{array}{r} 7552 \\ - \underline{3218} \end{array}$$

Subtract front end: $7000 - 3000 = 4000$ Estimate: $4000 +$

Do you want it closer? Adjust: $500 - 200 = 300$ Closer Estimate: $4300 +$

Sometimes renaming is involved in the estimate.

$$\begin{array}{r} 7452 \\ - \underline{3618} \end{array}$$

- Subtract front end: $7000 - 3000 = 4000$ Estimate: $4000 -$ (overestimate) (**NOTE:** The “-“ sign is used to denote that 4000 is an overestimate, thus, further estimates must be adjusted downwards, or “something else has to be subtracted from the estimate”. In other words, the actual answer is less than the initial estimate. (Whether or not we use this symbol and its attendant meanings will depend on how much you think it will help or confuse kids.)
- Look at the rest: 452 is less than 618 so we must rename 7452 to **6000 + 1400** $50 + 2$
 $1400 - 600 = 800$ and $6000 - 3000 = 3000$ Closer Estimate: 3800

b) Rounding Strategy

Rounding strategy focuses on the use of a set of specific rules to round off numbers up or down to the nearest 10, 100, 1000, etc.

For example:

$$\begin{array}{r} 832 \\ - \underline{367} \end{array}$$

- Round numbers up or down to nearest 10, 100, 1000, etc. (832 rounds down to 800 and 367 rounds up to 400)
- Subtract the rounded numbers. $800 - 400 = 400$ Estimate: 400

c) “Nice Numbers” Strategy

The focus of this strategy is to find, reorganize and/or round numbers so that they are easier to compute mentally.

For example:

$$\begin{array}{r} 5729 \\ - \underline{1815} \end{array}$$

- Round off a number to make subtraction easier. 1815 is close to 2000.
- Subtract. $5729 - 2000 = 3729$ Estimate: 3729

2.3.3 Multiplication

a) Front-end Strategy:

Front-end estimation focuses on computing the product where one of the factors is less than 10. The estimate can be refined or adjusted by rounding the digit of the next column.

For example:

$$6 \times 758$$

- Multiply front end. $6 \times 700 = 4200$ Estimate: 4200 + (underestimate)
- Do you want it closer? $6 \times 50 = 300$ $4200 + 300 = 4500$ Closer Estimate: 4500

b) Rounding:

Rounding focuses on computing the product of the two numbers, one or both of which have been rounded to the nearest 10, 100, 1000, etc. Because of the nature of multiplication, one needs to determine whether the result is an **underestimate** or an **overestimate**.

For example:

$$7 \times 285$$

- Round 285 to 300.
- Multiply $7 \times 300 = 2100$ Estimate: 2100 - (overestimate)

$$38 \times 51$$

- Round 38 to 40 and 51 to 50 $40 \times 50 = 2000$ Estimate: 2000 (Because one number is rounded up and the other rounded down, there is no general rule to refine the estimate.)

$$13 \times 42$$

- Round 13 to 10 and 42 to 40 $10 \times 40 = 400$ Estimate: 400 - (underestimate)

c) “Nice Numbers” Strategy

Much like rounding, this strategy endeavors to find numbers that are close to 10, 100.

For example:

$$38 \times 97$$

- Round off a number to make subtraction easier. 97 is close to 100.
- Multiply $38 \times 100 = 3800$ Estimate: 3800

2.3.4 Division

a) Initial Quotient Estimates:

This strategy focuses on determining the number of digits in the quotient (i.e., whether the quotient is in the 10s, 100s, 1000s, ...)

For example;

$$5/1281$$

- Determine the location of the quotient. (e.g., $5/1281 = 20?$, $200?$, $2000?$)
- Quotient has 3 digits, so must be in the hundreds. Initial Estimate: 200

b) Compatible Numbers:

This strategy focuses on selecting pairs of appropriate multiples. (sometimes referred to as “nice numbers”.)

For example:

$$6/257$$

- Find closest multiple for 6 (e.g., $6 \times 3 = 18$, $6 \times 4 = 24$, $6 \times 5 = 30$).

- Divide. $6/24 = 4$ Initial Estimate: 4 tens or 40⁺

$$63/437$$

- Round divisor. (63 rounds down to 60)
- Truncate divisor and dividend. $6/43$ Find closest multiple. $6 \times 7 = 42$ Initial Estimate: 7 tens or 70

2.3.5 Fractions

a) Benchmarks Strategy:

This strategy is used with fractions, where 0, $\frac{1}{2}$, and 1 are benchmarks.

For example:

$$2/5 + 7/8$$

- $2/5$ is close to $\frac{1}{2}$ and $7/8$ is close to 1 so $2/5 + 7/8$ is about $1 \frac{1}{2}$.

$$2 \frac{3}{8} + 4 \frac{6}{10}$$

- Add the whole numbers. $2 + 4 = 6$
- Estimate the sum of the fractional parts.
- $3/8 + 6/10$ is about 1, because each fraction is close to $\frac{1}{2}$ Estimate: 7

b) Rounding Strategy:

For example:

$$5 \frac{3}{4} - 2 \frac{7}{8}$$

- Round second number. $5 \frac{3}{4} - 3 = 2 \frac{3}{4}$ Estimate: $2 \frac{3}{4}$

c) Nice Numbers Strategy:

For example:

$$24 \frac{5}{8} + 18 \frac{1}{4} + 26 \frac{1}{2}$$

- Use “nice” whole numbers that are easy to add. $25 + 20 + 25 = 70$ Estimate: 70

d) Rounding Strategy:

For example:

$$8 \frac{1}{2} \times 6 \frac{1}{2}$$

- Round second number to whole number. $8 \times 6 = 48$ and $\frac{1}{2} \times 8 = 4$ Estimate: 52 (48 + 4)

OR

- Round both to whole numbers. $8 \times 6 = 48$ Estimate: 48

OR

- Round one up and one down. $8 \times 7 = 56$ OR $9 \times 6 = 54$ Estimates: 56 or 54

2.3.6 Decimals

a) Smaller/Larger Number Strategies:

The decimal portion of numbers between 0 and 10 is important when rounding. The decimal portion of larger numbers is less important because it contributes little value to the answer.

b) Front End Addition:

For example: **(smaller numbers)**

$$0.2 + 3.6 + 9.5 + 6$$

- Add whole number portion. $0 + 3 + 9 + 6 = 18$
- Estimate decimal portion. $0.2 + 0.6 + 0.5$ is between 1 and $1\frac{1}{2}$ Estimate: 19 or $19\frac{1}{2}$
OR
- Round, then add: $0 + 4 + 10 + 6 = 20$ Estimate: 20

c) Front End Subtraction:

For example **(smaller numbers)**

$$14.2 - 6.8$$

- Subtract whole number portion. $14 - 6 = 8$ Estimate: 8
OR
- Round off. $14 - 7 = 7$ Estimate: 7

For example

$$8.7 - 3.2$$

- Subtract whole number portion. $8 - 3 = 5$
- Subtract decimal portion. $0.7 - 0.2 = 0.5$ Estimate: 5.5

d) Nice Numbers Addition:

For example; (larger numbers)

$$47.2 + 38.5 + 41.7$$

- Use "nice numbers" to make adding easier. $50 + 40 + 40 = 130$ Estimate: 130

e) Front End or Rounding Subtraction:

For example **(larger numbers)**

$$\$541.03 - \$275.49$$

- Subtract front end digits. $500 - 200 = 300$ Estimate: \$300 -
OR
- Round then subtract. $500 - 300 = 200$ Estimate: \$200 +
OR
- Use nice numbers $550 - 250 = 300$ Estimate: 300 -

2.3.7 Multiplication and Division

Students can use many of the strategies for whole numbers and fractions.

For example:

a) Rounding:

$$286 \times 0.85$$

$$286 \times 1 = 286$$

Estimate: 286

b) Front-End:

$$4.3 \times 1.7$$

$$4 \times 1.5 = 4 \text{ and } 4 \times 0.5 = 2$$

Estimate: 6

c) Rounding:

$$72.65 \times 8.3$$

$$70 \times 8 = 560$$

Estimate: 560

d) Nice Numbers:

$$155.2 \div 37.6$$

$$160 \div 40 = 40$$

Estimate: 40

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3. Mental Math for Primary Children

Many mathematics curriculum documents and major mathematics associations recommend that educators pay close attention to various methods of computation, such as paper-and-pencil algorithms and calculators, with particular emphasis on the use of mental computation strategies. They point out the need to help young students develop an awareness of how to use basic facts with mental math to compute problems that involve larger numbers.

As noted in *Mental Math in the Primary Grades* (Reys1988):

...primary students ought to realize that a problem such as $40 + 50$ is simply an extension of the basic fact $4 + 5$ and thus be able to find the answer without using a paper-and-pencil algorithm.

Thus, good reasoning strategies help young students to learn and apply basic facts.

More globally, we want students to be proficient and comfortable with viewing, expressing, and representing numbers in different ways. Thus, we spend time to help students develop the ability to compose and decompose numbers, and to operate mentally on them in different ways.

Here are some of the more familiar strategies for mental computation. The list is not exhaustive by any means. In fact, each one of them can have variations invented by a young student! If you would like more details regarding these strategies, please refer to the resource mentioned above.

3.1 Some Mental Math Strategies

- a. **Counting on:** Use counting skills to find a sum.
 $8 + 2$ Think “8, nine, ten” so $8 + 2 = 10$
- b. **Doubling Numbers:** Double one of the numbers.
 - $4 + 5$ Think $4 + 4 + 1 = 9$
 - $12 - 5$ Think 5 and another 5 are 10 and add 2 makes 12 so $12 - 5 = 7$
- c. **Adding to make tens:** Think about combinations that make ten
 $8 + \underline{\quad} = 10$, $6 + \underline{\quad} = 10$
- d. **Adding with tens:** Adjust numbers to make 10.
 - 17 Think $10 + 7$
 - $8 + 6$ Think $10 + 4 = 14$
- e. **Numbers with Nine:** Use a nine to make multiples of ten.
 - $19 + 6$ Think add 1 to 19 and take 1 from 6 so $20 + 5 = 25$
 - $29 - 15$ Think add 1 to 29 and add 1 to 15 so $30 - 16 = 14$
 - $37 - 19$ Think add 1 to 19 and add 1 to 37 so $38 - 20 = 18$
- f. **Counting Back:** Subtract 1, 2, 3.
 $15 - 3$ Think “15, 14, 13, 12” so $15 - 3 = 12$
- g. **Counting Up:** Subtract 1, 2, 3.
 $9 - 7$ Think “9, 8, 7” so $9 - 7 = 2$
- h. **Thinking Addition:** Use a related addition fact.
 $12 - 5$ Think $5 + 7 = 12$ so $12 - 5 = 7$
- i. **Missing Parts:** Think addition (with 10s).
 $8 + \underline{\quad} = 14$ Think $8 + 2$ is 10 and add 4 makes 14, so $8 + 6 = 14$

j. Using Tens: Use addition to help with subtraction.

- $14 - 8$ Think $8 + 2$ is 10 and add 4 makes 14 so $14 - 8 = 6$
- $24 - 18$ Think $18 + 2$ is 20 and add 4 makes 24 so $24 - 18 = 6$

k. Calculators: Predict the next counting number(s).

- $+1 = = = =$ Think 1, 2, 3, 4, 5...
- $+5 = = = =$ Think 5, 10, 15, 20...
- $+10 = = = =$ Think 10, 20, 30, 40, 50...

l. 100 Chart: Use the chart to identify patterns.

There are many materials (teacher-made and commercial) that provide a wealth of useful strategies for predicting patterns and mentally computing sums and differences.

m. Chains: Use number sentences to practice multi-step computation.

- $+ 5 + 2 - 3 =$
- $30 - 10 + 5 + 20 =$

n. Multiples of Ten: Demonstrate the connection between basic facts and multiples.

- $30 + 50$ Think 3 tens plus 5 tens = 8 tens or 80
- $70 + 40$ Think 7 tens plus 4 tens = 11 tens or 110
- $40 + 35$ Think tens plus 3 tens plus 5 ones = 7 tens and 5 ones or 75
- $80 - 30$ Think 8 tens minus 3 tens = 5 tens or 50

o. Using 1: Add 1 to make multiples of 10.

- $36 + 49$ Think $36 + (49 + 1)$ or $36 + 50 - 1 = 85$
- $37 - 19$ Think $37 - (19 + 1)$ or $37 - 20 + 1 = 18$

p. Expanding addends: Write one of the numbers in expanded form.

$$35 + 23 \quad \text{Think } 35 + (20 + 3) = 55 + 3 = 58$$

q. Front End: Start with the lead digits to find the sum.

$$25 + 13 \quad \text{Think add 2 tens and 1 ten (3 tens), then add 5 ones and 3 ones (8 ones)} \\ \text{so } 25 + 13 = 38$$

r. Your Favourites: Please let me know if you have some favourites that don't appear above. I'll add them to the list.